BEHAVIOURAL APPROACH TO THE RISK MANAGEMENT

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Abstract
In this paper we discuss popular risk measures of risk, their attractions and limitations. In the first chapter, we consider classical approaches to risk management. There we can see the development of measures of risk and moreover, we consider the index options of statistical estimates of quantile measures of risk. On their analysis my diploma thesis was based. The main purpose was the formation of investment portfolio based on such combinations of measures of risk. In the second chapter, the future work is represented. There we present the prospect and cumulative prospect theories. We discuss again the previous models under behavioral finance framework. We propose a new measure based on Value at Risk by using assumptions of behavioral finance and try to make suggestions on other measures.
Background
Nowadays the analysis and estimation of market risks becomes one of the most actual problems at the formation of a portfolio of securities. Strengthening of economic intensity all over the world, instability and large-scale changes of economy force to improve methods of an estimation of risks.

For the first time portfolio theory has been mentioned in 1952 in the work of American mathematician-economist Harry Markowitz "Portfolio Selection". It has served as the beginning of prompt development and problem studying of optimization an investment portfolio.

Modern theory of investments developed, trying to correct the lacks inherent in model suggested by Markowitz. The theory of the follower of Markowitz - William Sharpe became one of alternative approaches. He has suggested dividing risk of a financial active on two components: market risk of a portfolio and own risk of a portfolio.

Besides change in the approach to the risk, the model of Sharpe simplifies a method of a choice of an optimum portfolio; the problem of square-law optimization (model of Markowitz) is reduced to the linear.

For variety of different financial technologies on management of risk and the profitableness, as a theoretical basis the model of an estimation of profitableness of the shares serves. It has been developed by William Sharpe, John Lintner and Jan Mossin independently in the mid-sixties of XX century and it is called CAPM (Capital Asset Pricing Model). Within the limits of this approach profitableness of shares depending on behavior of the market is considered as a whole and it is supposed that investors make decisions, considering only expected profitableness and risk.

Another equilibrium pricing model – Arbitrage Pricing Theory serves as an alternative of CAPM. It has been created in the beginning of 80-th years by Stefan Ross. The basic assumption of the given model that each investor tries to use
possibility of increase in profitableness of the portfolio without risk increase. The mechanism promoting given possibility, is an arbitrage. The arbitrage is a reception of riskless profits at use of the different prices for identical production or securities. In real life of the arbitrage doesn't exist, as it is the ideal situation assuming obligatory reception of profit at absence of risk of loss of investments. The main difference between CAPM and APT is that the last one takes more than one factor into consideration.

More seriously the question on necessity of management of risks has risen owing to a train of large financial crashes in the early nineties.

After a series of scandalous ruins of such giants as Orange Country, Barings, Metallgesellschaft, Daiwa, economists were convinced once again that the large sums of money can be lost owing to underestimation of importance of control and absence of adequate mechanisms of management of financial risks. Many financial institutions were engaged in researches in the field of a risk management. As a result, the middle of 90th years was marked by two important events in the theory and practice of the application of the financial markets: wide use of quantile measures of risk VaR (Value-at-Risk), that was offered for the first time by the company J.P.Morgan as alternatives of a dispersion dominating before as a risk measure, and allocation of axiomatic classes of measures of risk, first of which was the class of coherent measures of the risk, entered by Artzner, Delbaen, Eber and Heath. Then other classes of measures of risk (convex, additive, revolted, limited to an average) have been entered also.

The measure that was suggested by J.P.Morgan was derived from a system based on standard portfolio theory. It was using estimates of the standard deviations and various correlations between the returns to different traded instruments. Other financial institutions that were also working on their own internal models considered VaR systems that were not based on Portfolio Theory. For example, there were VaR systems that were built using a historical simulation approach that estimate VaR from a histogram of past profit and loss data for the institution as a
Another system was developed by using method Monte-Carlo. It is much more sophisticated and powerful than previous two systems.

The measure of risk VaR is used widely not only by securities houses and investment banks, but also by commercial banks, pension funds and other financial institutions, and non-financial corporate. It is recommended for applying by Basel Committee on Banking Supervision.

One of the shortcomings of VaR is that all VaR systems are backward-looking. All of them use past data for future forecasting. But there is no confidence that the past relationships will continue in the future. For example, the one of the difficulties that we can have is a market crash. In these cases, we should remain aware of the limitation and supplement VaR analysis with scenario analyses that tell us what we might lose under hypothetical circumstances.

Secondly, all VaR systems are based on the assumptions that may not be valid in real situations. However, the correct action will be aware of limitations and behave accordingly.

The measure of risk VaR ignores the weight of tails of distribution of profitablenesses (or losses) – doesn't consider possible big profits (losses) which can occur to small probabilities. One more essential shortcoming: VaR isn't a coherent measure; in particular, it doesn't possess property of subadditivity. It is possible to result examples, when VaR a portfolio more than sum VaR of parts of portfolio of which it consists. It contradicts common sense. Really, if to consider a risk measure as the size of the capital reserved for a covering of market risk for a covering of risk of all portfolio there is no necessity to reserve more than the sum of reserves of components of portfolio.

VaR encourages trading strategy which gives the good income at the majority of scenarios, but can sometimes lead to catastrophic losses.

S. Uryasev  has been offered a coherent measure of the risk Conditional Value-at-Risk (CVaR). CVaR is the expectation of income, lower than VaR. This method of risk assessment can take into account the huge losses that can occur with small
probability. While this measure of risk does not become common in risk assessments.

Conditional Value at Risk (CVaR) is also called expected shortfall, average value at risk (AVaR), and expected tail loss (ETL). The "expected shortfall at q% level" is the expected return on the portfolio in the worst q% of the cases. For high values of q CVaR ignores the most profitable but unlikely possibilities, for small values of q it focuses on the worst losses. On the other hand, unlike the discounted maximum loss even for lower values of q expected shortfall does not consider only the single most catastrophic outcome. A value of q often used in practice is 5%. CVaR estimates the value (or risk) of investments in conservative manner, focusing on the less favorable results.

CVaR has two important properties. Firstly, it is coherent measure of risk. Secondly, it is a spectral measure of financial portfolio risk. It requires a quantile-level q, and is defined to be the expected loss of portfolio value given that a loss is occurring at or below the q-quantile. CVaR could be easily decomposed and optimized, while VaR is not. Moreover, CVaR requires a larger size of sample than VaR for the same level of accuracy.

In my diploma thesis I considered two index options of statistical estimates quantile measures of risk (VaR-CVaR). The main purpose was to develop the methodology for formation of the investment portfolios. For achievement necessary results n portfolios were generated by using the gage of pseudo-random numbers of Uichman-Hill. After that the values of measures of risk \( M_1, M_2 \) for all generated portfolios on time interval T were calculated. We investigate two new measures of risk based on left and right tails of distribution:

\[
M_1(\alpha, \beta) = \frac{\text{Var}^R_{\beta}(x)}{\text{Var}^L_{1}(x)},
\]

\[
M_2(\alpha, \beta) = \frac{\text{CVaR}^R_{\alpha}(x)}{\text{CVaR}^L_{\alpha}(x)}.
\]
Then we assume that $P_1(X) \leq P_2(X) \leq \ldots \leq P_T(X)$ - sequence of profitableness of a portfolio ordered on increase $X$ for $T$ days of supervision, $l = \alpha T$.

Then

$$VaR_\alpha(X) = P_1(X), \quad VaR_\alpha^+(X) = P_{T-l+1}(X)$$

$$CVaR_\alpha(X) = \frac{\sum_{i=1}^{l} P_i(X)}{l}, \quad CVaR_\alpha^+(X) = \frac{\sum_{i=1}^{l} P_{T-l+1}(X)}{l}$$

So,

$$M_1(\alpha, \beta) = \frac{P_1(X)}{P_{T-l+1}(X)}$$

$$M_2(\alpha, \beta) = \frac{\sum_{i=1}^{l} P_i(X)}{\sum_{i=1}^{l} P_{T-l+1}(X)}$$

The next step was to make a choice among those portfolios for which the risk measures $M_1$, $M_2$ are maximum at each value of parameter $\alpha$. Parameter was chosen from the interval $[0.01;0.05]$ with the value of the step equal to 0.01. After that profitableness was calculated on a time interval $\tau$ for the portfolios chosen before. Time horizons $T$ and $\tau$ were equal to 300 days and to 10 days accordingly. The last step was finding of value of parameter $\alpha$ at which the maximum profitableness was received.

Computing experiment was carried out on 5 sets of shares of Russian companies.

There were taken 31 companies circulating on Russian stock exchanges.

In each set were included the shares of 10 issuers.

There were considered two periods of time: before crisis (July 2006 – December 2007) and the crisis (January 2008 – May 2009).

Finally, as a result we received that the use of index measures based on the measure CVaR, leads to the formation of a more effective portfolio, rather than
at the use of measure based on the VaR during the crisis, and on the contrary, the use of an index measure of risk based on the measure VaR preferable to the use of an index measure of risk based on the least CVaR during it. In the absence of a crisis it is recommended to use \( \alpha=0,01 \); during the crisis the highest profitableness will be reached with \( \alpha=0,03 \) and \( \alpha=0,04 \).
Behavioral finance and Value at Risk

Laboratory and real-life experiments have shown that not always decision-makers behave in a rational manner. Owing to limitations in processing information, agents often make choices different from what should be made given information available. Psychological biases arising from this process could be:

- Cognitive bias, which is the judgmental distortion due to cognitive factors (cognitive dissonance, anchoring, framing, etc.)
- Emotional bias, distortion due to emotional factors (greed, fear, etc.)

In practice, there have been found many anomalies which are inconsistent with traditional finance settings. Some notable of them in financial markets are:

- The value premium puzzle (price-earnings ratio effect): historical outperformance of value stocks over growth stocks;
- Size premium puzzle (small firm effect): historically, small capitalization stocks have outperformed large capitalization stocks;
- Equity premium puzzle: historically, equity has outperformed virtually default-free debt (government bonds) (Mehra and Prescott (1985)).
- Under-reaction or over-reaction of stock prices to earnings announcement: stocks with higher past returns are overvalued and those with lower past return are undervalued (De Bondt and Thaler (1985)).
- Asset allocation puzzle: investors don’t hold the same composition of risky assets (Canner, Mankiw and Weil (1997)).
- Price fluctuations with no information: Harris and Gurel (1986) found that there is an average abnormal return of 3.5% for stocks added to S&P 500. Another emblematic example is the stock market crash on Monday, 19, 1987.

Attempts have been made to explain anomalies met in decision making under uncertainty.

We can recall the Bernoulli method in solving the St Petersburg paradox by a decreasing marginal utility of wealth. We also recall the rank-dependent model
which, in order to explain Allais paradox, generalizes the expected utility by applying a cumulative probability weighting function. The most satisfactory explanation of decision making is given by the cumulative prospect theory. Following the concept of irrational behavior, Kahneman and Tversky posited the Prospect Theory. Unlike the expected utility theory of Von Neumann and Morgenstern, which is a normative paradigm, the prospect theory is a descriptive pattern of decision making.

According to this theory, the decision process follows two steps:

- Editing phase
- Evaluation phase

The editing phase is the preliminary investigation. Prospects are reformulated. Outcomes and probabilities are transformed. The main operations during the editing stage are:

- Coding, outcomes are perceived as gains and losses with respect to some reference point
- Combination, simplification by combining probability with identical outcomes
- Segregation, separation of the riskless component from the risky one
- Cancellation, discarding common components to the prospects
- Simplification, rounding outcomes and probabilities. Discarding highly uncertain outcomes
- Detection of dominance, dominated prospects are discarded

After the editing phase, which mostly alters preferences, prospects are evaluated. During the evaluation stage, weights are assigned to probabilities and values are assigned to outcomes.

The prospect theory also accounts for many other situations like the shift of reference point, observed risk attitudes and cases in which probabilities are not provided (in these cases weights are linked to specific events).

Later, the same authors proposed an extension of the prospect theory called the Cumulative Prospect Theory. The main features of the new version are:

- Weights are linked to cumulative probabilities. So, it is applicable to uncertain as
well as to risky prospects with continuous distribution. It is coherent with stochastic dominance.

- Different decision weights for gains and losses

We know that, in classical finance, returns are usually modeled on one risk factor (recall the diffusion process from which the famous Black-Scholes formula is derived). Moreover, it is often assumed that the risk factor has some stability properties through time hence allowing the application of statistical inference. Unfortunately, it is also well known that most of models fail in practice. Very often many estimation techniques assume that the future will be the same like the past, even parametric methods (like Montecarlo Simulation) still rely to some extent on the past to derive their parameters. Although other attempts have been made to improve the modeling of the behavior of asset prices (let mention mixed process like jump diffusion), we argue that they remain simply descriptive approaches. Their predictive power is still arguable mostly due to uncertainty of misperception. A behavioral approach is very interesting because it segregates prices into two background factors. The advantage is huge as it helps to explain many classical theory inconsistencies found in practice and can help to better control the behavior of asset prices (at least from a risk management viewpoint).

Let call $p_t$ the price of the risky asset at time $t$, $\xi_{t+1}$ noise trader misperception of time $t+1$ risky asset price at time $t$, $n$ - the proportion of noise traders, $\alpha_{ns}$ the share of the unsafe asset hold by the noise traders, $d_{t+1}$ the real dividend per share at time $t+1$, $r$ - the real risk free rate. On occasion, just for theoretic, we will replace time $t$ by today; time $t+1$ by tomorrow or next period and time $t-1$ by yesterday or last period.

By definition, noise traders are investors mistaking noise for information Kyle (1985) and Black (1986); and arbitrageurs are investors with correct beliefs. Let’s consider price and return behaviors in a market with noise traders. Now we are going to check what that involves from a risk management viewpoint.

We assume that all risk factors determining the return are bounded, we mean, with finite means and variances. Without this assumption, as we’ve seen in the thought
experiment, the traditional risk measurement process is useless. Moreover we assume that the return has standard distribution $\mathcal{N}(\mu, \sigma)$, $\alpha$ is the confidence level, $\beta_{t+1}$ is the tomorrow estimated mean and $\delta_{t+1}$ is the tomorrow estimated standard deviation from a sample with length $m$.

The parametric Value at Risk is

$$VaR_{t+1} = -\mu_{t+1} + \delta_{t+1}^{-1}(1 - \alpha).$$

We propose the following adjusted market risk measure

$$VaR_{t+1} = -\mu_{t+1} + \delta_{t+1}^{-1}(1 - \alpha) + \theta_{t+1}(R_{t+1}, I)$$

Where $\theta_{t+1}(R_{t+1}, I)$ means that the systematic misperception depends on the exposure ($R_{t+1}$), current and future information (I). Normally it includes also volatility uncertainty’s contribution to the return, since this component behaves in the risk measure like the expected return from misperception. But we assumed it is negligible just to highlight the effect of systematic noise trading.

$$\theta_{t+1}(R_{t+1}, I) = \frac{(1 + r)\Delta_t n_{t+2} z_{t+2} + r(1 + r)n_{t+2} \bar{x}_{t+2} - r(1 + r)n_{t+3} \bar{x}_{t+1} - 2\Delta_{t+2} n_{t+2} z_{t+2} \delta_{t+2}}{\bar{n}_t}$$

Contrary to the efficient market hypothesis, it is obvious that the risk measure would depend both on fundamentals and investors’ misperception. This hints us to analyze more in detail the goodness of VaR under a market with noise traders.

We will get adjusted market risk measure based on VaR. After that, we can see its implication to our traditional risk measurement process. We assume an estimation based on the method of moments. After making all calculations and receiving results we can compare the VaR estimator and the VaR.
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